Trigonometrija 1

Stavovi:

$$\sin(\frac{\pi}{2} - x) = \cos x \tag{1}$$

$$\sin(-x) = -\sin x \tag{2}$$

$$\sin(\pi - x) = \sin x \tag{3}$$

$$\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2} \tag{4}$$

$$\sin x - \sin y = 2\sin\frac{x-y}{2}\cos\frac{x+y}{2} \tag{5}$$

$$\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2} \tag{6}$$

$$\cos(x-y) = -2\sin\frac{x-y}{2}\sin\frac{x+y}{2} \tag{7}$$

$$\sin(x+y) = \sin x\cos y + \cos x\sin y \tag{8}$$

$$\sin(x-y) = \sin x\cos y - \cos x\sin y \tag{9}$$

$$\cos(x+y) = \cos x\cos y - \sin x\sin y \tag{10}$$

$$\cos(x-y) = \cos x\cos y + \sin x\sin y \tag{11}$$

Primjer1. Odrediti

$$\frac{1 - 4\sin 10^\circ \cdot \sin 70^\circ}{2\sin 10^\circ} \tag{12}$$

rjesenje:

$$\frac{1 - 4\sin 10^\circ \cdot \sin 70^\circ}{2\sin 10^\circ} = \frac{1 - 2(\cos 60^\circ - \cos 80^\circ)}{2\sin 10^\circ} = \frac{1 - 1 + 2\cos 80^\circ}{2\sin 10^\circ} = \frac{2\sin 10^\circ}{2\sin 10^\circ} = 1 \tag{13}$$

Primjer 2. Pokazati da je

$$\cos\frac{\pi}{7} + \cos\frac{5\pi}{7} + \cos\frac{3\pi}{7} = \frac{1}{2} \tag{14}$$

rjesenje:

$$\cos\frac{\pi}{7}+\cos\frac{5\pi}{7}+\cos\frac{3\pi}{7}=\frac{\sin\frac{\pi}{7}\cos\frac{\pi}{7}+\sin\frac{\pi}{7}\cos\frac{5\pi}{7}+\sin\frac{\pi}{7}\cos\frac{3\pi}{7}}{\sin\frac{\pi}{7}}= \tag{15}$$

$$=\frac{\sin\frac{2\pi}{7}+\sin\frac{6\pi}{7}-\sin\frac{4\pi}{7}+\sin\frac{4\pi}{7}-\sin\frac{2\pi}{7}}{2\sin\frac{\pi}{7}}=\frac{\sin\frac{6\pi}{7}}{2\sin\frac{\pi}{7}}=\frac{1}{2} \tag{16}$$

Primjer 3. Pokazati da je

$$\tan^2 10^\circ+\tan^2 50^\circ+\tan^2 70^\circ=9 \tag{17}$$

rjesenje:

Neka je

$$\tan 10^\circ=t \tag{18}$$

Tada imamo

$$\tan 50^\circ=\tan(60^\circ-10^\circ)=\frac{\sqrt{3}-t}{1+\sqrt{3}t} \tag{19}$$

$$\tan 70^\circ=\tan(60^\circ+10^\circ)=\frac{\sqrt{3}+t}{1-\sqrt{3}t} \tag{20}$$

$$t^2+(\frac{\sqrt{3}-t}{1+\sqrt{3}t})^2+(\frac{\sqrt{3}+t}{1-\sqrt{3}t})^2=6+9\cdot\frac{(3t-t^3)^2}{(1-3t^2)^2}=6+9\tan^2 30^\circ=9 \tag{21}$$

Primjer 4. Pokazati da je

$$(4\cos^2 9^\circ-1)(4\cos^2 27^\circ-1)(4\cos^2 81^\circ-1)(4\cos^2 243^\circ-1)=1 \tag{22}$$

rjesenje:

$$(4\cos^2 9^\circ-1)(4\cos^2 27^\circ-1)(4\cos^2 81^\circ-1)(4\cos^2 243^\circ-1)= \tag{23}$$

$$=\frac{\sin 9^\circ(4\cos^2 9^\circ-1)(4\cos^2 27^\circ-1)(4\cos^2 81^\circ-1)(4\cos^2 243^\circ-1)}{\sin 9^\circ}= \tag{24}$$

$$=\frac{\sin 27^\circ(4\cos^2 27^\circ-1)(4\cos^2 81^\circ-1)(4\cos^2 243^\circ-1)}{\sin 9^\circ}=\frac{\sin 81^\circ(4\cos^2 81^\circ-1)(4\cos^2 243^\circ-1)}{\sin 9^\circ}= \tag{25}$$

$$=\frac{\sin 243^\circ(4\cos^2 243^\circ-1)}{\sin 9^\circ}=\frac{\sin 729^\circ}{\sin 9^\circ}=\frac{\sin 9^\circ}{\sin 9^\circ}=1 \tag{26}$$

Primjer 5. Pokazati da je

$$\tan 15^\circ \tan 25^\circ \tan 35^\circ = \tan 5^\circ \tag{27}$$

rjesenje:

$$\tan x \cdot \tan(60^\circ - x) \cdot \tan(60^\circ + x) = \tan(3x) \tag{28}$$

Koristeci (13) je

$$\tan 15^\circ \tan 25^\circ \tan 35^\circ = \tan 15^\circ \cdot \frac{\tan 25^\circ \cdot \tan 35^\circ \tan 85^\circ}{\tan 85^\circ} = \tan 15^\circ \cdot \frac{\tan 75^\circ}{\tan 85^\circ} = \frac{1}{\tan 85^\circ} = \tan 5^\circ \tag{29}$$

Zadaci za samostalan rad

1.Pokazati da je

$$\sec^4 \frac{\pi}{7} + \sec^4 \frac{2\pi}{7} + \sec^4 \frac{3\pi}{7} = 416 \tag{30}$$

2.Pokazati da je

$$\cot^2 \frac{\pi}{7} + \cot^2 \frac{2\pi}{7} + \cot^2 \frac{3\pi}{7} = 5 \tag{31}$$

3.Pokazati da je

$$\sec \frac{\pi}{11} + \sec \frac{3\pi}{11} + \sec \frac{5\pi}{11} + \sec \frac{7\pi}{11} + \sec \frac{9\pi}{11} = 6 \tag{32}$$

4.Odrediti

$$\frac{\cot^3 75^\circ + \tan^3 75^\circ}{\cot 75^\circ + \tan 75^\circ} \tag{33}$$

5.Pokazati da je

$$\tan 70^\circ + \tan 10^\circ - \tan 50^\circ = \sqrt{3} \tag{34}$$

6.Pokazati da je

$$\tan \frac{3\pi}{11} + 4 \sin \frac{2\pi}{11} = \sqrt{11} \tag{35}$$

7.Pokazati da je

$$8 \sin^4 10^\circ - 8 \sin^3 10^\circ - 6 \sin^2 10^\circ + 7 \sin 10^\circ = 1 \tag{36}$$

8.Pokazati da je

$$\cos\frac{2\pi}{21}+\cos\frac{8\pi}{21}+\cos\frac{10\pi}{21}=\frac{\sqrt{21}+1}{4} \tag{37}$$

9.Pokazati da je

$$\cos\frac{2\pi}{13}+\cos\frac{6\pi}{13}+\cos\frac{8\pi}{13}=\frac{\sqrt{13}-1}{4} \tag{38}$$

10. Pokazati da je

$$\tan 50^\circ+\tan 60^\circ+\tan 70^\circ=\tan 80^\circ \tag{39}$$

11. Pokazati da je

$$\tan^2 10^\circ+\tan^2 50^\circ+\tan^2 70^\circ=9 \tag{40}$$

12. Pokazati da je

$$\tan^2 36^\circ+\tan^2 72^\circ=10 \tag{41}$$

13. Pokazati da je

$$\tan^4 36^\circ+\tan^4 72^\circ=90 \tag{42}$$

14. Pokazati da je

$$\tan^6 36^\circ+\tan^6 72^\circ=850 \tag{43}$$

15. Pokazati da je

$$\tan^8 36^\circ+\tan^8 72^\circ=8050 \tag{44}$$

16. Pokazati da je

$$\tan^4 4^\circ+\tan^4 8^\circ+\tan^4 12^\circ+...+\tan^4 88^\circ=990 \tag{45}$$

17. Pokazati da je

$$\sin\frac{\pi}{14}\cdot\sin\frac{2\pi}{14}\cdot\sin\frac{3\pi}{14}\cdot\sin\frac{4\pi}{14}\cdot\sin\frac{5\pi}{14}\cdot\sin\frac{6\pi}{14}=\frac{\sqrt{7}}{64} \tag{46}$$

18. Pokazati da je

$$4\sin\frac{2\pi}{5}+\tan\frac{2\pi}{5}=5\cot\frac{\pi}{5} \tag{47}$$